Ejercicio 5 – Convocatoria Ordinaria 1 – Curso 2021/22

Calcular la solución mínimo-cuadrática del sistema de ecuaciones lineales:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix} = 1 - 1 + 0 - 0 - 0 - 0 = 0$$

$$\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow rg(A) = 2$$

$$A|b = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1 + 0 + 0 - 0 - 1 + 0 = -2 \neq 0$$

$$\Downarrow$$

$$rg(A|b) = 3$$

Como $rg(A) = 2 \neq 3 = rg(A|b) \Rightarrow$ Sist. incompatible.

Solución mínimo-cuadrática es la solución

$$A^t A x = A^t b$$

$$A^t A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$A^t b = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{array}{r} 2x+y+z=1 \\ x+2y-z=1 \\ x-y+2z=0 \end{array}\right\}$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 8-1-1-2-2-2=0$$

$$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 \neq 0 \Rightarrow rg(A^tA)=2$$

$$\Downarrow$$

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$$z=\lambda$$

$$\begin{array}{r} 2x+y=1-\lambda \\ -2(x+2y=1+\lambda) \\ \hline -3y=-1-3\lambda \end{array} \Rightarrow y=\frac{1}{3}+\lambda$$

$$x=1+\lambda-2\left(\frac{1}{3}+\lambda\right)=$$

$$=-\lambda+\frac{1}{3}$$

Solución: $\left(-\lambda+\frac{1}{3},\ \frac{1}{3}+\lambda,\ \lambda\right)$